PROPENSITIES TO CONSUME AND THE PERSONAL DISTRIBUTION OF INCOME IN A SIMPLE KEYNESIAN MODEL

by Fabio Ravagnani*

As is known, explicit consideration of the different propensities to consume of the various groups of income earners plays a major role in the attempts to extend Keynes's and Kalecki's principle of effective demand to the analysis of growth and functional distribution. This paper points out, by means of a simple multi-sectoral model, that those different consumption habits contribute to shaping the personal distribution of income in the economy, not only directly but also by influencing the impact on personal distribution, of policy measures involving income transfers.

Com'è noto, l'esplicita considerazione delle differenti propensioni al consumo dei vari gruppi di percettori di reddito svolge un ruolo rilevante nei tentativi di estendere il principio della domanda effettiva di Keynes e Kalecki all'analisi della crescita e della distribuzione funzionale. Nel presente lavoro si mostra, mediante un semplice modello multisettoriale, che quelle diverse abitudini di consumo contribuiscono a conformare la distribuzione personale del reddito nell'economia, non solo direttamente ma anche influenzando l'impatto sulla distribuzione personale, di misure di politica che comportino trasferimenti di reddito.

1. Introduction

As is known, explicit consideration of the different propensities to consume of the various groups of income earners plays a major role in the attempts to extend Keynes's (1936) and Kalecki's (1939) principle of effective demand to the analysis of growth and functional distribution (for example, Kaldor, 1956; Robinson, 1962; Rowthorn, 1981; Badhuri, Marglin, 1990; for a comprehensive account of the developments in the field, cf. Lavoie, 2014). The aim of this paper is to point out, by means of a simple multi-sectoral model, that those different consumption habits contribute to shaping the personal distribution of income in the economy.

The paper is organised as follows. Section 2 describes the economic system on which the analysis is based and shows how a well-defined personal distribution of income can be associated with the generic equilibrium position of the economy. Section 3 then uses that

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construction to discuss the direct impact on personal distribution of the propensities to consume of the distinct groups of income earners. Section 4 goes on to highlight an indirect link between consumption habits and personal distribution. In particular, it points out that the differences in the consumption habits of income earners contribute to determining the impact on personal distribution of policy measures involving income transfers. Finally, Section 5 recapitulates and briefly comments on the limitations of the analysis presented.

2. The economic system taken into consideration

2.1. Basic features

Our discussion will be based on a Keynesian model of the economy with exogenous investment decisions, in which the money wages and the monetary prices of commodities are taken as given and constant. By assumption, three industries are active in the economy, which respectively produce two different consumption goods (1 and 2) and a capital good (good 3). As regards production conditions, we assume that the capital good is not subject to wear and tear and that only labour and the capital good are employed as inputs in all industries. Moreover, we assume that, while the production of goods 1 and 3 requires a single type of labour (say, labour of type *A*), the production of good 2 requires a combination of labour of type *A* and labour of type *B*, with the latter performing auxiliary tasks and receiving a lower wage. Finally, we assume that the labour cost for each euro's worth of output is lower in industry 2 than in industry 1, which implies that the short-period profits for each euro's worth of product will be higher in the former industry (short-period profits are defined as the difference between the monetary value of output and the total labour cost). Regarding distribution, we shall distinguish between three separate groups of income earners: the workers of type *A* and those of type *B*, who live entirely on their wages, and a given number H^{π} of households who do not work for a wage and live on their individual shares of the profits generated in the economy. In order to determine the personal distribution of income associated with the generic equilibrium position of the economy, we shall assume that the whole amount of short-period profits generated in industry is uniformly allocated among the H^{π} pure profit earners. We shall further assume the profit earners constitute the richest part of the population and, finally, that the three groups of income earners have different consumption habits. In particular, it will be assumed that:

(i) the workers of type *B* (henceforth workers *B*) do not save and spend their whole income on good 1;

(ii) the workers of type *A* (henceforth workers *A*) do not save and spend their income partly on good 1 and partly on good 2;

(iii) the H^{π} profit earners spend a fraction of their income on good 2 and save the remaining part.

The last assumption can be seen as a variant of the usual post-Keynesian distinction between the propensity to consume out of wages and the propensity to consume out of profits, characterised by the presence of wage earners who belong to separate income classes and, for this reason, display different consumption patterns.[1]

[1] Those patterns can in turn be justified by assuming that, at the exogenously given prices and wages, (a) workers *B* are forced to spend their whole income on good 1 in order to provide for their subsistence, and (b) workers *A* choose to

2.2. The model

The economy just outlined can be formally represented through the following equations:

$$Y_1 = C_1 \qquad (1)$$

$$Y_2 = C_2 \qquad (2)$$

$$Y_3 = I \qquad I > 0 \qquad (3)$$

$$C_1 = c_1^A \, w \,(L_1^A + L_2^A + L_3^A) + w^B L^B \qquad 0 < c_1^A < 1 \qquad (4)$$

$$C_2 = C_2^A + C_2^\pi \qquad (5)$$

$$C_2^A = c_2^A \, w \,(L_1^A + L_2^A + L_3^A) \qquad c_2^A = 1 - c_1^A \qquad (6)$$

$$C_2^\pi = c^\pi (\Pi_1 + \Pi_2 + \Pi_3) \qquad 0 < c^\pi < 1 \qquad (7)$$

$$\Pi_i = Y_i - w L_i^A \qquad i = 1, 3 \qquad (8)$$

$$\Pi_2 = Y_2 - w L_2^A - w^B L^B \qquad (9)$$

$$w^B = \gamma w \qquad 0 < \gamma < 1 \qquad (10)$$

$$L_i^A = l_i \, Y_i \qquad i = 1, 2, 3 \qquad (11)$$

$$L^B = L_2^A \qquad (12)$$

$$l_2 = 0.5 \, l_1 \qquad (13)$$

In the first three equations, Y_1, Y_2, Y_3 denote the values of sectoral outputs calculated at the assumed given and constant monetary prices, and C_1, C_2, I the corresponding total demands in terms of money. Equation (4) states that the demand for good 1 comes entirely from the $L^A = L_1^A + L_2^A + L_3^A$ workers *A* employed in the three industries – who receive the money wage w and spend the fraction c_1^A of their income on that good – and

spend a fraction of their higher income on good 2 with a view to imitating the consumption style of the rich. Within this framework, it is conceivable that external factors (e.g. publicity campaigns) may alter the decisions on the part of workers *A* concerning the amount of income to be spent on good 2. (Similar imitation effects, triggered e.g. by interaction with the elites of other countries, may conceivably alter the consumption decisions of the profit earners.) With regard to the assumptions just introduced, it should be noted that the approach to consumption decisions based on hierarchies of needs, income classes with their respective conventions, imitation effects, and rules of thumb such as propensities to consume is a characteristic feature of post-Keynesian analysis (cf. Lavoie, 2014, Ch. 2). Moreover, basic elements of this approach are adopted by scholars adhering to the modern reappraisal of the Classical theory of value (for example, Schefold, 1997, Ch. 14). A comparative assessment of the approach under discussion *vis-a-vis* the dominant one based on innate individual preferences and intertemporal utility maximisation is beyond the scope of this paper. We limit ourselves to pointing out that empirical studies aimed at testing post-Keynesian models with differentiated propensities to consume are available in the literature and should be taken into account (cf., for instance, Onaran, Galanis, 2012 and the references given there).

from the L^B workers *B* employed in industry 2, who receive the money wage w^B. Equation (6) states that workers *A* spend the fraction c_2^A of their income on good 2 and do not save. Equation (7) assumes that the H^π profit earners spend the fraction c^π of their income on good 2 and save the remaining part. Equations (8)-(9) define the amounts of short-period profits obtained in the three industries, on the assumption that industry 2 employs both types of workers. Equation (10) assumes that the money wage for workers *B* is a given fraction γ of that paid to workers *A*. Equation (11) assumes that the quantity l_i of labour *A* is required in the production of a euro's worth of good *i* ($i = 1, 2, 3$). Equation (12) postulates that the two types of workers are employed in the fixed proportion 1:1 in industry 2. Finally, equation (13) assumes that the quantity of labour *A* required in the production of a euro's worth of good 2 is one half of that required in the production of a euro's worth of good 1. Considered jointly with the equations (9)-(12), equation (13) implies that the amount of profits obtainable in industry 2 for each euro's worth of product is $\pi_2 = [(1+\gamma)/2]\, l_1\, w$, and is therefore larger than the analogous amount of profits $\pi_1 = 1 - l_1\, w$ obtainable in industry 1.

By assumption, the money wage *w* is taken as given together with I, γ, c_1^A, c^π, l_1, l_3. System (1)-(13) is therefore determined. In order to complete the model, we introduce the profitability conditions

$$1 - l_i\, w > 0 \qquad\qquad i = 1, 3 \qquad\qquad (P)$$

Together with the strictly positive investment demand postulated in (3), which ensures strictly positive equilibrium outputs in all industries, the conditions (*P*) guarantee that the solution to the system is economically meaningful.

2.3. Equilibrium positions and the personal distribution of income

For any appropriate specification of the 'data' w, I, γ, c_1^A, c^π, l_1, l_3 – complying, in particular, with the conditions (*P*) – system (1)-(13) determines the equilibrium values of sectoral outputs Y_i^* with the corresponding employment levels L_i^{A*}, L^{B*} and amounts of profits Π_i^* ($i = 1, 2, 3$). Ignoring for simplicity the distinction between individual agents and households, and taking the available labour force *LF* as exogenously given, we will be able to associate with the equilibrium position of the economy a well-defined personal distribution of income. More precisely, we will conclude that in the equilibrium position:
(i) H^π households receive the individual income $m^{\pi*} = (\sum_i \Pi_i^*)/H^\pi$;
(ii) $L^{A*} = \sum_i L_i^{A*}$ households receive the individual income *w*;
(iii) L^{B*} households receive the individual income w^B;
and, finally,
(iv) $U^* = LF - (L^{A*} + L^{B*})$ households, being unemployed, earn no income at all.

Remark. The assumed consumption habits require that the individual income of the profit earners be larger than *w*. Throughout the paper we shall refer to economies fulfilling this requirement.

3. The consumption habits of the distinct groups of income earners and their impact on personal distribution

We shall now use the theoretical construction just presented to examine the direct impact on personal distribution of the consumption habits of the distinct groups of income earners.

3.1. The consumption habits of workers A

We shall begin our examination by illustrating, through a numerical exercise, the impact on personal distribution of the consumption habits of workers *A*. The exercise is structured as follows. We shall construct two numerical specifications of the model and assume that they refer to two distinct economies, α and β. In those specifications, we shall assign the same values to all parameters and exogenous variables, except for those attributed to the propensity c_1^A. Then we shall calculate and compare the equilibrium positions of the two economies with the associated personal distributions of income.[2]

EXERCISE 1. Assume that the following conditions hold in both economy α and economy β:
$I = 10{,}000$; $w = 100$; $\gamma = 0.8$; $l_1 = 0.009$; $l_3 = 0.008$; $c^\pi = 0.7$;
$H^\pi = 50$; $LF = 3{,}000$.
Moreover, assume that the value of c_1^A is 0.95 in α and 0.90 in β. Calculation of the equilibrium position and the corresponding personal distribution for both economies yields the following results.

ECONOMY α ($c_1^A = 0.95$)	ECONOMY β ($c_1^A = 0.90$)
$Y_1^* = 245{,}710.2$	$Y_1^* = 225{,}026.6$
$Y_2^* = 35{,}591.1$	$Y_2^* = 46{,}477.2$
$Y_3^* = 10{,}000$	$Y_3^* = 10{,}000$
$L_1^{A*} = 2{,}211$	$L_1^{A*} = 2{,}025$
$L_2^{A*} = 160$	$L_2^{A*} = 209$
$L_3^{A*} = 80$	$L_3^{A*} = 80$
$L^{B*} = 160$	$L^{B*} = 209$

PERSONAL DISTRIBUTION (Economy α)

	Number	Individual income
Profit earners	50	666.6
Workers *A*	2,451	100
Workers *B*	160	80
Unemployed	389	0

Gini index: 0.219

PERSONAL DISTRIBUTION (Economy β)

	Number	Individual income
Profit earners	50	666.6
Workers *A*	2,314	100
Workers *B*	209	80
Unemployed	477	0

Gini index: 0.251

In the above exercise, the postulated small difference in the consumption habits of workers *A* gives rise to appreciable differences in the two equilibrium positions. In

[2] The equilibrium positions will be calculated on the basis of the mathematical solutions for Y_1, Y_2 as emerging from system (1)-(13) (for these solutions, see the Appendix, Part III). The same procedure will be used in the second exercise to be presented in this section.

particular, we see that economy β, characterised by lower propensity to consume good 1 on the part of those workers, displays: (a) lower employment in industry 1 accompanied by larger employment in industry 2, and (b) higher unemployment level.

Let us now examine the associated differences in the personal distribution of income. As regards the upper part of the distribution, we see that the individual income of the profit earners is exactly the same in the two economies. This is a consequence of the assumption that only the profits earners save, which implies that the total income of that group of households must amount to $\Sigma_i \Pi_i^* = [1/(1-c^\pi)]I$ in equilibrium independently of the workers' consumption habits. As for the other parts, we may regard them for a moment as if they referred to the same economy observed over two consecutive periods α and β. Adopting this point of view, the passage from α to β can be seen as the outcome of the dismissal of a certain number of workers in industry 1, with some of the dismissed workers being immediately absorbed by industry 2 (half as workers A and half as workers B) and the remaining part reaching the ranks of the unemployed. In view of the latter considerations, and considering that the upper part of the distribution is the same, it might be conjectured that economy β displays higher income inequality. This conjecture is confirmed by the Gini index, which is appreciably higher in β.

We shall now argue that the results emerging from Exercise 1 are not accidental but can be generalised. To begin with, it should clearly appear that the higher inequality observed in β is the consequence of the aforementioned differences (a)-(b) in sectoral employment levels. The following proposition points out that analogous differences will be found in any hypothetical reformulation of the exercise.

PROPOSITION 1. *Consider two specifications of the model under consideration in which all parameters and exogenous variables are the same except for the parameter defining the consumption habits of workers A. Then in equilibrium, the specification characterised by lower value of* c_1^A *will display a smaller number of workers in industry 1, a larger number of workers in industry 2, and a higher unemployment level.*
PROOF: see the Appendix, Part I.

As a further step in our argument, let us now consider: given two specifications of the model that differ exclusively for the consumption habits of workers A, under which conditions will the differences in equilibrium employment levels be such that the economy with lower value of c_1^A displays higher inequality as in Exercise 1? An answer is provided by Proposition 2.

PROPOSITION 2. *Consider a first specification of the model such that the following conditions hold:*

$$l_1 \geq l_3,\ 0.5 \leq \gamma < 1,\ L^{B*} \geq H^\pi \qquad (C)$$

Then any alternative specification in which c_1^A *has a lower value than in the first and all the other parameters and exogenous variables have the same values as in the first will be associated with a higher value of the Gini index.*
PROOF: see the Appendix, Part II.

It should be noted that economy α of the numerical exercise belongs to the class of economies complying with the conditions (C).

3.2. The consumption habits of the profit earners

Let us now move on to examine the impact on personal distribution of the propensity to consume of the pure profit earners. We shall start with an exercise similar to that previously discussed, in which the key difference between the economies to be compared lies in the values assigned to c^{π}.

EXERCISE 2. Assume that the following conditions hold in both economy α and economy δ:
$I = 10{,}000$; $w = 100$; $\gamma = 0.8$; $l_1 = 0.009$; $l_3 = 0.008$; $c_1^A = 0.95$;
$H^{\pi} = 50$; $LF = 3{,}000$.
Moreover, assume that the value of c^{π} is 0.7 in α and 0.72 in δ. Calculation of the equilibrium position and the corresponding personal distribution for both economies yields the following results.

ECONOMY α ($c^{\pi} = 0.7$)	ECONOMY β ($c^{\pi} = 0.72$)
$Y_1^* = 245{,}710.2$	$Y_1^* = 263{,}348.8$
$Y_2^* = 35{,}591.1$	$Y_2^* = 38{,}838.9$
$Y_3^* = 10{,}000$	$Y_3^* = 10{,}000$
$L_1^{A*} = 2{,}211$	$L_1^{A*} = 2{,}370$
$L_2^{A*} = 160$	$L_2^{A*} = 175$
$L_3^{A*} = 80$	$L_3^{A*} = 80$
$L^{B*} = 160$	$L^{B*} = 175$

PERSONAL DISTRIBUTION (Economy α)

	Number	Individual income
Profit earners	50	666.6
Workers *A*	2,451	100
Workers *B*	160	80
Unemployed	389	0

Gini index: 0.219

PERSONAL DISTRIBUTION (Economy β)

	Number	Individual income
Profit earners	50	714.3
Workers *A*	2,625	100
Workers *B*	175	80
Unemployed	200	0

Gini index: 0.165

In Exercise 2, the slight difference in the propensity to consume of profit earners engenders appreciable differences in the equilibrium employment levels of the two economies. In particular, we see that economy δ, characterised by greater value of c^{π}, displays larger employment of both workers *A* and workers *B*.

As regards personal distribution, we see that the individual income of the profit earners is greater in δ.[3] At the same time, focusing on the opposite part of the distribution, we find that the number of households with no income is considerably smaller in δ. In

[3] This is a further consequence of the assumption that only the profit earners save, which, as noted earlier in the text, implies that the total profits must amount to $\Sigma_i \Pi_i^* = [1/(1-c^{\pi})]\, I$ in equilibrium.

these circumstances, the effect on income inequality of the postulated difference in the consumption habits of the profit earners is not obvious. To assess this point, we must resort to the Gini index, which indicates that the economy characterised by the greater value of c^{π} displays lower inequality in the personal distribution of income.

We shall now argue that also the results emerging from Exercise 2 can be generalised. To begin with, inspection of the solutions to the model indicates that differences in equilibrium employment levels analogous to those observed in economy α and in economy δ will be found in any reformulation of the exercise (see the Appendix, Part III). Having established this point, we can ask the following: given two specifications of the model that differ exclusively for the value of c^{π}, under which conditions will the differences in equilibrium employment levels be such that the specification with greater value of that propensity displays lower inequality as in Exercise 2? An answer is provided by Proposition 3.

PROPOSITION 3. *Consider a first specification of the model under consideration such that the conditions (C) listed in Proposition 2 are fulfilled. Then any alternative specification in which the value of c^{π} is larger than in the first and all the other parameters and exogenous variables have the same values as in the first, will be associated with a lower value of the Gini index.*
PROOF: see the Appendix, Part IV.

4. DIFFERENTIATED CONSUMPTION HABITS AND THE IMPACT OF POLICY MEASURES INVOLVING INCOME TRANSFERS

We will now highlight an indirect link between consumption habits and personal distribution of income. More precisely, we shall point out that the differences in the propensities to consume of income earners contribute to determining the impact on personal distribution of policy measures involving income transfers. This will be done by examining two distinct cases.

4.1. Case 1: a subsidy to firms financed by taxing workers A

The examination of this case will start with a numerical exercise. We will first define the initial equilibrium position of the economy. Then we will assume that a policy measure is introduced into the system, which consists of levying a small tax of τ euros on the wage of the workers A and using the total receipts T to subsidise one or more of the industries in existence. (On the stipulated assumption that profits are wholly allocated to a given group of households, the measure boils down to an income transfer from workers A to that group.) Finally, we shall calculate the new equilibrium of the economy after the introduction of the measure and discuss the role played by the differentiated consumption habits of income earners as regards the observed changes in personal distribution.

The aforementioned equilibrium positions will be determined on the basis of this modified version of the model of Section 2, henceforth referred to as *MV1*:

$$Y_1 = C_1 \tag{1}$$

$$Y_2 = C_2 \tag{2}$$

$$Y_3 = I \qquad I > 0 \qquad (3)$$

$$C_1 = c_1^A\ (w-\tau)\ (L_1^A + L_2^A + L_3^A) + w^B\ L^B \qquad 0 < c_1^A < 1,\ 0 \leq \tau < w - w^B \qquad (4')$$

$$C_2 = C_2^A + C_2^\pi \qquad (5)$$

$$C_2^A = c_2^A\ (w-\tau)\ (L_1^A + L_2^A + L_3^A) \qquad c_2^A = 1 - c_1^A \qquad (6')$$

$$C_2^\pi = c^\pi\ (\Pi_1 + \Pi_2 + \Pi_3 + T) \qquad 0 < c^\pi < 1 \qquad (7')$$

$$T = \tau\ (L_1^A + L_2^A + L_3^A) \qquad (8')$$

plus the last six equations of the original model and the conditions (*P*).[4]

The exercise will thereby consist of taking an initial solution to *MV1* with $\tau = 0$ and then calculating the corresponding new solution for $\tau > 0$.[5] In commenting on the numerical results, it will be tacitly assumed that the economy converges to the new equilibrium. That assumption will be justified in the last part of this subsection.

Exercise 3. Assume that the economy is initially in equilibrium under the following conditions:

$I = 10{,}000$; $w = 100$; $g = 0.8$; $l_1 = 0.009$; $l_3 = 0.008$; $c_1^A = 0.90$; $c^\pi = 0.7$; $H^\pi = 50$; $LF = 3{,}000$, $\tau = 0$.

Then suppose that a tax of $\tau = 0.5$ euros is introduced on the wage of workers *A* and used to subsidise the productive sector, with all other conditions remaining the same. Calculation of the initial equilibrium with $\tau = 0$ and the new equilibrium with $\tau = 0.5$ yields the following results.

Initial equilibrium ($\tau = 0$)	New equilibrium ($\tau = 0.5$)
$Y_1^* = 225{,}026.6$	$Y_1^* = 215{,}772$
$Y_2^* = 46{,}477.2$	$Y_2^* = 45{,}488.5$
$Y_3^* = 10{,}000$	$Y_3^* = 10{,}000$
$L_1^{A*} = 2{,}025$	$L_1^{A*} = 1{,}942$
$L_2^{A*} = 209$	$L_2^{A*} = 205$
$L_3^{A*} = 80$	$L_3^{A*} = 80$
$L^{B*} = 209$	$L^{B*} = 205$

[4] Note that the second inequality attached to the equation (4') postulates that the tax be so small that the individual disposable income of workers *A* remains larger than w^B, as is required for justifying the different propensities attributed to the two types of workers.

[5] Both numerical solutions will be calculated on the basis of the mathematical solutions for Y_1, Y_2 as emerging from the modified model under consideration (for these solutions, see the Appendix, Part V).

Personal distribution	Number	Individual income
Profit earners	50	666.6
Workers *A*	2,314	100
Workers *B*	209	80
Unemployed	477	0

Gini index: 0.251

Personal distribution	Number	Individual disposable income
Profit earners	50	666.6
Workers *A*	2,227	99.5
Workers *B*	205	80
Unemployed	568	0

Gini index: 0.280

We see that the measure under examination gives rise to an appreciable fall in the number of workers employed in industry 1 and a slight fall in the number of workers employed in industry 2. As regards the changes in personal distribution, note that the upper part is not affected at all by the measure, as it was to be expected given the assumption that only the profit earners save.[6] As to the other parts, note that the dimension of the intermediate income classes is smaller in the new equilibrium owing to lower employment in the first two industries and, moreover, that the individual income in the richer of those classes is slightly lower due to the assumed taxation. In these circumstances, it might be conjectured that the new equilibrium will be characterised by higher income inequality. This conjecture is confirmed by the Gini index, which is appreciably higher in the new equilibrium.

From the foregoing discussion it clearly appears that the higher inequality observed in the new equilibrium originates, albeit not exclusively, from the adjustments in sectoral employment that take place after the introduction of the measure. Considering then that these adjustments occur precisely because the consumption habits of workers *A* differ from those of the profit earners, we conclude that the differentiated propensities to consume of income earners play a role as regards the impact of the policy measure on income inequality. This point will be further examined later on.

Let us now generalise the results emerging from the numerical exercise. With regard to the changes in employment levels, the following proposition can be stated.

Proposition 4. *Consider an initial equilibrium position of MV1 with* $\boldsymbol{\tau} = 0$ *and a new equilibrium position in which all exogenous variables and parameters are the same as in the initial specification except for the strictly positive value attributed to* $\boldsymbol{\tau}$. *Then the new equilibrium with* $\boldsymbol{\tau} > 0$ *will display lower employment in both industry 1 and industry 2.*
Proof: see the Appendix, Part V.

As to the conditions ensuring that the introduction of the measure and the related changes in employment lead to higher inequality, the following proposition can be stated.

Proposition 5. *Consider an initial equilibrium position of MV1 with* $\boldsymbol{\tau} = 0$ *and such that the following conditions hold:*

[6] That assumption implies that the aggregate disposable income of the profit earners must amount to $M^{\pi *} = [1/(1- c^{\pi})]I$ in equilibrium independently of the value of $\boldsymbol{\tau}$.

$$l_1 \geq l_3,\ 0.5 \leq \gamma < 1,\ H^{\pi} \leq \frac{l_2 c^{\pi} I}{1 - c^{\pi}} \qquad (C')$$

Moreover, consider a new equilibrium position in which all exogenous variables and parameters are the same as in the initial specification except for the strictly positive value attributed to τ. *Then the new equilibrium with* $\tau > 0$ *will be associated with a higher value of the Gini index.*

Proposition 5 is proved in the Appendix, Part VI. It should be noted that the third inequality in (C'), which may seem obscure, basically ensures that the condition $L^{B*} > H^{\pi}$ holds in both the initial and the new equilibrium.[7] Note also that the economy of Exercise 3 complies with the conditions (C').

It is now time to deal with a point that has merely been postulated so far, namely the convergence to the new equilibrium. As soon as the subsidy and the associated taxation are introduced, the economy, which was originally in equilibrium with $\tau = 0$, will display excess demand or supply in the markets for consumption goods (while the original equilibrium on the capital good market will not be upset, as the measure only affects consumers' demand). As a consequence, the outputs of industry 1 and industry 2 will adjust. Thus assume that those outputs adjust in continuous time according to the process

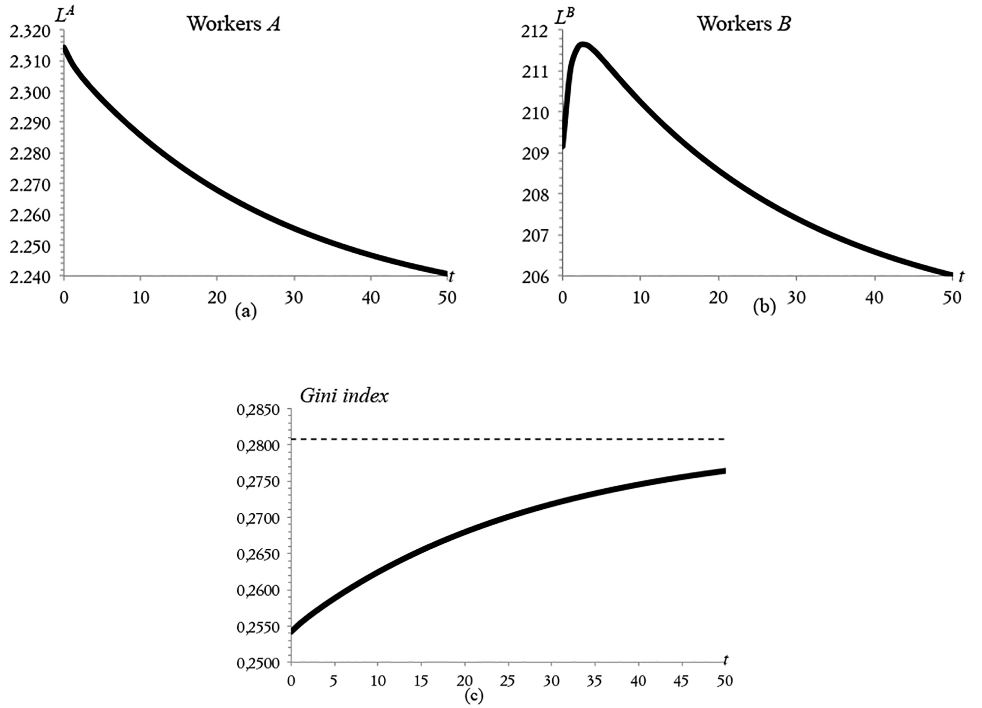
$$Y_1^{'}(t) = ED_1(t),\ Y_2^{'}(t) = ED_2(t) \qquad (*)$$

where $Y_i^{'}(t)$ is the rate of output adjustment that prevails in industry i at the generic instant t and $ED_i(t)$ is the rate of excess demand for good i as prevailing at t ($i = 1, 2$).[8] By analysing this process, which translates into a simple system of differential equations, it can be proved that after the introduction of the measure the economy will asymptotically converge to the corresponding new equilibrium (see the Appendix, Part VII). Within this context, Propositions 4-5 can therefore be used to assess the effects of the policy measure under discussion. To conclude on this point, we can examine the adjustment to the new equilibrium in the case of the economy of Exercise 3, focusing in particular on the changes in employment levels induced by the process (*) and on the related changes in the Gini. The adjustment paths of these variables, calculated by means of *Mathematica*, are represented in Figure 1.

[7] Recall that the aggregate disposable income of the profit earners amounts to $M^{\pi *} = [1/(1\text{-}c^{\pi})]I$ in the generic equilibrium of the economy. This means that the number of workers B required for meeting the profit earners' consumption demand must be $\overline{L}^{B} = l_2\,[c^{\pi}/(1\text{-}c^{\pi})]I$ in equilibrium, and is therefore determined by exactly the expression on the right-hand side of the third inequality in (C'). Considering then that the total number of workers B must exceed $\overline{L}^{B}$ in equilibrium, since also workers A consume good 2, we see that the third inequality in (C') entails $L^{B*} > H^{\pi}$ at any admissible value of τ.

[8] In process (*) we are implicitly taking as the physical unit of commodity i the quantity of i that is worth one euro ($i = 1, 2$). With this choice of physical units, both the physical sectoral outputs and the physical demanded quantities of the different commodities numerically coincide with their respective monetary values. Moreover, the physical units just defined do not change in the course of process (*) since the monetary prices of commodities are constant by assumption. As a consequence, the variables $Y_1^{'}(t)$, $Y_2^{'}(t)$ in (*) can be seen as the adjustment rates of physical outputs, to be equated to the corresponding physical excess demands.

Figure 1. The adjustments in employment levels (a, b) and the ensuing time path of the Gini index (c) in the economy of Exercise 3



We see that once the measure has been introduced, the adjustments in employment levels prompted by the differentiated consumption habits of income earners originate a progressive rise of the Gini index. We could accordingly say that those adjustments act as an *inequality multiplier* in the economy.

4.2. Case 2: a subsidy to workers B financed by taxing the profit earners

Let us now examine a second policy measure, starting from the following exercise. After defining the initial equilibrium of the economy, we shall assume that a subsidy of τ euros in favour of workers *B* is introduced in the system and financed by taxing the pure profit earners. Then we shall calculate the new equilibrium after the introduction of the subsidy and discuss the effects of the measure on personal distribution, on the tacit assumption that the economy converges to the new equilibrium.

The equilibrium positions to be compared will be determined on the basis of this modified version of the model of Section 2, henceforth referred to as *MV2*:

$$Y_1 = C_1 \tag{1}$$

$$Y_2 = C_2 \tag{2}$$

$$Y_3 = I \qquad I > 0 \qquad (3)$$

$$C_1 = c_1^A\, w\,(L_1^A + L_2^A + L_3^A) + (w^B + \tau)\, L^B \qquad 0 < c_1^A < 1,\ 0 \le \tau < w - w^B \qquad (4")$$

$$C_2 = C_2^A + C_2^\pi \qquad (5)$$

$$C_2^A = c_2^A\, w\,(L_1^A + L_2^A + L_3^A) \qquad c_2^A = 1 - c_1^A \qquad (6)$$

$$C_2^\pi = c^\pi\,(\Pi_1 + \Pi_2 + \Pi_3 - T) \qquad 0 < c^\pi < 1 \qquad (7")$$

$$T = \tau\, L^B \qquad (8")$$

plus the equations (8)-(13) of the original model and the conditions (*P*).[9]

Exercise 4. Assume that the economy is initially in equilibrium under the following conditions:
I = 10,000; w = 100; γ = 0.8; l_1 = 0.009; l_3 = 0.008; c_1^A = 0.95; c^π = 0.7; H^π = 50; LF = 3,000; τ = 0.

Then suppose that a subsidy of τ = 5 euros to every worker *B* is introduced and financed by taxing the profit earners, with all other conditions remaining the same. Calculation of the initial equilibrium with τ = 0 and the new equilibrium with τ = 5 yields the following results.

Initial equilibrium ($\tau = 0$)	New equilibrium ($\tau = 5$)
$Y_1^* = 245{,}710.2$	$Y_1^* = 253{,}144.9$
$Y_2^* = 35{,}591.1$	$Y_2^* = 35{,}933.4$
$Y_3^* = 10{,}000$	$Y_3^* = 10{,}000$
$L_1^{A*} = 2{,}211$	$L_1^{A*} = 2{,}278$
$L_2^{A*} = 160$	$L_2^{A*} = 162$
$L_3^{A*} = 80$	$L_3^{A*} = 80$
$L_2^{B*} = 160$	$L_2^{B*} = 162$

Personal distribution ($\tau = 0$)

	Number	Individual income
Profit earners	50	666.6
Workers *A*	2,451	100
Workers *B*	160	80
Unemployed	389	0

Gini index: 0.219

Personal distribution ($\tau = 5$)

	Number	Individual disposable income
Profit earners	50	666.6
Workers *A*	2,520	100
Workers *B*	162	85
Unemployed	318	0

Gini index: 0.194

[9] More precisely, the numerical equilibrium positions will be calculated on the basis of the mathematical solutions for Y_1, Y_2 as emerging from the version of the model under consideration (for these solutions, see the Appendix, Part VIII).

We see that the measure originates an appreciable increase in the number of workers *A* employed and a slight increase in the employment of workers *B*. As regards personal distribution, note that the upper part is not influenced by the measure.[10] Moreover, note that the dimension of the intermediate income classes is larger in the new equilibrium owing to the increase in employment and, finally, that the individual income in the poorer of those classes is slightly greater owing to the assumed small subsidy. In these circumstances, it might be conjectured that the new equilibrium will be characterised by lower income inequality. The reported values of the Gini confirm that this is indeed the case.

The foregoing remarks indicate that the lower inequality observed in the new equilibrium originates, though not exclusively, from the adjustments in employment levels taking place after the introduction of the subsidy and the associated taxation. Since these adjustments occur just because the consumption habits of workers *B* differ from those of the profit earners, we conclude that the differentiated propensities to consume of income earners contribute to determining the impact on inequality of the measure under discussion. This contribution will shortly be examined in more detail.

We may go on now to generalise the results emerging from the exercise. The propositions that follow address, respectively, the effect of the measure on employment levels and the conditions ensuring that the introduction of the measure and the ensuing employment changes lead to lower inequality.

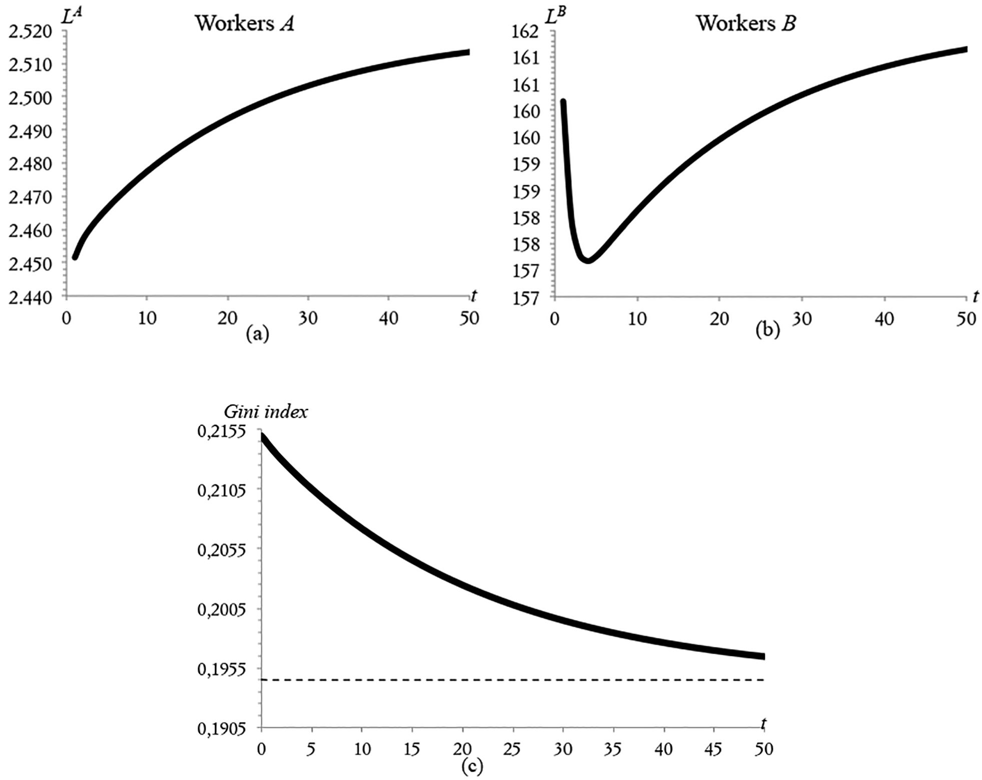
PROPOSITION 6. *Consider an initial equilibrium position of MV2 with* $\boldsymbol{\tau} = 0$ *and a new equilibrium position in which all exogenous variables and parameters are the same as in the initial specification except for the strictly positive value attributed to* $\boldsymbol{\tau}$. *Then the new equilibrium with* $\boldsymbol{\tau} > 0$ *will display a larger number of workers employed in both industry 1 and industry 2.*
PROOF: see the Appendix, Part VIII.

PROPOSITION 7. *Consider an initial equilibrium position of MV2 with* $\boldsymbol{\tau} = 0$ *and such that the conditions (C) of Proposition 2 hold. Moreover, consider a new equilibrium position in which all exogenous variables and parameters are the same as in the initial specification except for the strictly positive value attributed to* $\boldsymbol{\tau}$. *Then the new equilibrium with* $\boldsymbol{\tau} > 0$ *will be associated with a lower value of the Gini index.*
PROOF: see the Appendix, Part IX.

Let us finally deal with the convergence to the new equilibrium that has been taken for granted so far. Thus assume that once the subsidy and the related taxation have been introduced, the outputs of industry 1 and industry 2 adjust in continuous time according to the process (*) already mentioned in the treatment of Case 1. In the Appendix, Part X, we examine that process as applied to *MV2* and show that, after the introduction of the measure, the economy asymptotically converges to the corresponding new equilibrium. To conclude on this aspect, let us briefly examine the adjustments in employment levels taking place in the economy of Exercise 4 under process (*), with the associated changes in the Gini index. The adjustment paths of these variables, calculated by means of *Mathematica*, are represented in Figure 2.

[10] As in Case 1, this is a consequence of the assumption that savings come entirely from the profit earners.

Figure 2. The adjustments in employment levels (a, b) and the ensuing time path of the Gini index (c) in the economy of Exercise 4



We see that, after the introduction of the measure, the adjustments in employment levels prompted by the differentiated propensities to consume give rise to a progressive fall of the Gini index. We could therefore say that those adjustments act as an *equality multiplier* in the economy.

5. Conclusions

It has been argued in this paper that the different propensities to consume of income earners, insofar as they influence sectoral production and employment levels, contribute to shaping the personal distribution of income in the economy. This point has been discussed on the basis of a Keynesian model with three industries and three categories of income earners, in which it is possible to associate a well-defined personal distribution to the generic equilibrium position of the economy. Within this framework, we first examined the direct impact on personal distribution of the consumption habits of two distinct groups of income earners. It was shown that the propensities to consume of these groups influence in a foreseeable way not only the characteristics of income classes but, given certain conditions, also the degree of income inequality as measured by the Gini index. Then we pointed out that the differences in the propensities to consume of income earners contribute to determining the impact on personal distribution of policy measures involving income transfers. To highlight this aspect, we examined two distinct, hypothetical measures. In both cases, it was shown that, once the measure is introduced, the differences in consumption habits trigger changes in employment that alter the dimensions of income classes in a foreseeable way. Moreover, it was shown that, given certain conditions, the introduction of the measure and the ensuing employment changes alter the value of the Gini index in a well-defined direction.

The aforementioned results are admittedly conditioned by the features of the model on which the discussion has been based, which include not only the assumptions that savings come exclusively from the profit earners and that investment decisions are exogenously given, but also specific hypotheses concerning, for instance, the proportions in which workers *A* and *B* are required for the production of the different consumption goods. On the other hand, the analysis put forward suggests a method of examining the links between consumption habits and personal distribution that seems applicable to less restrictive representations of the economy. The natural extension of the discussion developed in this paper will therefore consist of assessing to what extent the results summarised above continue to hold when the specific hypotheses introduced at this stage are gradually relaxed.

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Appendix

I. Proof of Proposition 1

1. Let ε and η be two specifications of the model that differ exclusively for the value of c_1^A. Assume that η is the specification with lower value of c_1^A and focus on the equilibrium positions associated with the two specifications. Since the total profits must be the same in those positions, the sum of the profits generated in the first two industries, $\Pi_{12}^* = \Pi_1^* + \Pi_2^*$, must also be the same.

2. Now denote by $Y_{i\varepsilon}^*$, i = 1, 2, the equilibrium outputs in the first two industries of the economy ε and by $Y_{i\eta}^*$, i = 1, 2, the corresponding equilibrium outputs in the economy η. Since ε and η differ exclusively for the value of c_1^A, it is impossible that $Y_{1\varepsilon}^* = Y_{1\eta}^*$ and $Y_{2\varepsilon}^* = Y_{2\eta}^*$ may simultaneously hold. Moreover, since Π_{12}^* is the same in both equilibria, either $Y_{1\eta}^* > Y_{1\varepsilon}^*$, $Y_{2\eta}^* < Y_{2\varepsilon}^*$ or $Y_{1\eta}^* < Y_{1\varepsilon}^*$, $Y_{2\eta}^* > Y_{2\varepsilon}^*$ must hold.

3. Then consider the differences $\Delta Y_1 = Y_{1\eta}^* - Y_{1\varepsilon}^*$, $\Delta Y_2 = Y_{2\eta}^* - Y_{2\varepsilon}^*$. Since Π_{12}^* is the same in both equilibria, the following equality must hold:

$$(1 - l_1 w)\ \Delta Y_1 + \left(1 - \frac{1+\gamma}{2} l_1 w\right)\ \Delta Y_2 = 0 \qquad \text{(I.1)}$$

where $(1 - l_1 w)$ is the amount of profits obtainable in industry 1 for each euro's worth of production and $\left(1 - \frac{1+\gamma}{2} l_1 w\right)$ is the analogous amount of profits obtainable in industry 2. Equality (I.1) in turn entails

$$\Delta Y_1 = -\left(\frac{1 - \frac{1+\gamma}{2} l_1 w}{1 - l_1 w}\right) \Delta Y_2 \qquad \text{(I.2)}$$

where the bracketed term is greater than 1 since $0 < \gamma < 1$. Denoting that term by k we obtain

$$\frac{\Delta Y_1}{\Delta Y_2} = -k \qquad k > 1 \qquad \text{(I.2')}$$

4. Now consider the differences in equilibrium employment levels $\Delta L_i^A = L_{i\eta}^{A*} - L_{i\varepsilon}^{A*}$, $i = 1, 2$. Since $\Delta L_i^A = l_1\ \Delta Y_1$, $\Delta L_2^A = l_2\ \Delta Y_2 = (l_1/2)\,\Delta Y_2$, the following equality must hold:

$$\frac{\Delta L_1^A}{\Delta L_2^A} = 2\ \frac{\Delta Y_1}{\Delta Y_2} \qquad \text{(I.3)}$$

Taking (I.2') into account, from (I.3) we obtain

$$\Delta L_1^A = -2\,k\ \Delta L_2^A \qquad k > 1 \qquad \text{(I.4)}$$

5. Finally, recall that it was established in step 2 that either $\Delta Y_1 > 0$, $\Delta Y_2 < 0$ or $\Delta Y_1 < 0$, $\Delta Y_2 > 0$ must hold. The former case can now be ruled out, however, as in view of (I.4) it would imply larger employment of workers A in economy η, which in turn, since c_1^A is lower in η, would give rise to larger demand for good 2 in that economy, thereby contradicting $\Delta Y_2 < 0$. Thus it can be concluded that $\Delta Y_1 < 0$, $\Delta Y_2 > 0$ must hold, which in turn implies ΔL_1^A

<0, ΔL_2^A >0 and, in view of (I.4) and the assumed equality $L_2^B = L_2^A$, larger unemployment in the economy η.

II. Proof of proposition 2

1. Consider the well-known formula for the Gini index

$$G = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N}|y_i - y_j|}{2N^2\mu}$$

where y_i (y_j) is the income of the individual i (j), N is the total number of individuals in the population and μ is the mean income. In the case of the model of Section 2, that formula can be rewritten as follows:

$$G = \frac{1}{N}\left[LF - wl_1 \frac{\gamma L^B(L^B + 2L^A + H^\pi) + L^A(L^A + H^\pi)}{L^B + L^A + (l_1 - l_3)I}\right] \tag{II.1}$$

where L^A, L^B denote the number of workers A and B employed in the economy, LF is the available labour force, H^π is the number of profit earners and $N = LF + H^\pi$.

2. For fixed values of w, I, γ, l_1, l_3, H^π and LF, formula (II.1) yields the value of the Gini index as a well-defined function of L^A and L^B that we denote $G(L^A, L^B)$. The partial derivatives of the function are:

$$\frac{\partial G}{\partial L^A} = -\frac{wl_1}{N}\,\frac{[2(\gamma L^B + L^A) + H^\pi](l_1 - l_3)I + \gamma(L^B)^2 + 2L^B L^A + (L^A)^2 + (1-\gamma)H^\pi L^B}{[L^B + L^A + (l_1 - l_3)I]^2}$$

$$\frac{\partial G}{\partial L^B} = -\frac{wl_1}{N}\,\frac{[2\gamma(L^B + L^A) + \gamma H^\pi](l_1 - l_3)I + \gamma(L^B)^2 + (2\gamma - 1)(L^A)^2 + 2\gamma L^B L^A + (\gamma - 1)H^\pi L^A}{[L^B + L^A + (l_1 - l_3)I]^2}$$

It is readily ascertained that for $l_1 \geq l_3$, $0.5 \leq \gamma < 1$, $L^B \geq H^\pi$ the following inequalities simultaneously hold:

$$\frac{\partial G}{\partial L^A} < 0,\ \frac{\partial G}{\partial L^B} < 0,\ \frac{\partial G / \partial L^A}{\partial G / \partial L^B} > 1 \tag{II.2}$$

3. Now consider that, for fixed values of w, I, γ, l_1, l_3, H^π, LF and c^π, the equilibrium employment levels L^{A*}, L^{B*} can be represented as functions of c_1^A. We can accordingly write $L^{A*} = L^A(c_1^A)$, $L^{B*} = L^A(c_1^A)$. From Proposition 1 we see that the following inequalities must hold for $c_1^A \in (0, 1)$:

$$\frac{dL^A}{dc_1^A} > 0,\ \frac{dL^B}{dc_1^A} < 0,\ \frac{d(L^A + L^B)}{dc_1^A} > 0 \tag{II.3}$$

which in turn imply

$$-\frac{dL^B / dc_1^A}{dL^A / dc_1^A} < 1 \tag{II.4}$$

4. Finally note that for fixed values of w, I, γ, l_1, l_3, H^π, LF and c^π, the value of the Gini index in the equilibrium position of the economy, which we denote by G^*, can be represented as a function of c_1^A:

$$G^* = G^*(c_1^A) = G\,[L^A(c_1^A), L^A(c_1^A)]$$

Thus consider an economy with $c_1^A = \overline{c}_1^A$ and such that the conditions (*C*) of Proposition 2 are fulfilled. In the equilibrium position of that economy, both the last inequality in (II.2) and the inequality (II.4) necessarily hold. As a consequence, the following inequality must also be fulfilled:

$$\frac{\partial G / \partial L^A}{\partial G / \partial L^B} > -\frac{dL^B / dc_1^A}{dL^A / dc_1^A} \tag{II.5}$$

Taking into account the signs of the partial derivatives as indicated in (II.2)-(II.3), from (II.5) we obtain

$$\frac{\partial G}{\partial L^A}\frac{dL^A}{dc_1^A} + \frac{\partial G}{\partial L^B}\frac{dL^B}{dc_1^A} = \frac{dG^*}{dc_1^A} < 0 \qquad \text{for } c_1^A = \overline{c}_1^A \tag{II.6}$$

Now note that since $\frac{dL^B}{dc_1^A} < 0$ for $c_1^A \in (0, 1)$, the conditions (*C*) continue to hold at any admissible value of c_1^A lower than $\overline{c}_1^A$. As a consequence, the argument put forward to assess the sign of $\frac{dG^*}{dc_1^A}$ for $c_1^A = \overline{c}_1^A$ can be repeated for $c_1^A \in (0, \overline{c}_1^A)$. We thus conclude that $\frac{dG^*}{dc_1^A} < 0$ for $c_1^A \in (0, \overline{c}_1^A]$, which means that any alternative equilibrium position obtained by attributing to c_1^A lower value than $\overline{c}_1^A$, while keeping the values of all other parameters unchanged, will be associated with a higher value of the Gini index.

III. Solutions to the model of Section 2

$$Y_1^* = \frac{wl_1(\gamma + c_1^A)}{2 - wl_1[1 + c_1^A + (1 - c_1^A)\gamma wl_1]}\left[\frac{c^\pi}{1 - c^\pi} + \frac{(1 - c_1^A)wl_3}{1 - c_1^A wl_1}\right] I + \left[\frac{c_1^A wl_3}{1 - c_1^A wl_1}\right] I$$

$$Y_2^* = \frac{2(1 - c_1^A wl_1)}{2 - wl_1[1 + c_1^A + (1 - c_1^A)\gamma wl_1]}\left[\frac{c^\pi}{1 - c^\pi} + \frac{(1 - c_1^A)wl_3}{1 - c_1^A wl_1}\right] I$$

It clearly emerges from these equations that a greater value of c^π entails a larger equilibrium production in both industry 1 and industry 2 and, therefore, larger employment of workers *A* and *B*.

IV. Proof of proposition 3

1. We shall follow the same method as in Part II. Thus let us return to the function $G(L^A, L^B)$ as defined in that part. We have seen that $\frac{\partial G}{\partial L^A} < 0$, $\frac{\partial G}{\partial L^B} < 0$ when the conditions (*C*) hold.

2. Then consider that for fixed values of w, I, γ, l_1, l_3, H^π, LF and c_1^A, the equilibrium employment levels L^{A*}, L^{B*} can be represented as functions of c^π. We can accordingly write $L^{A*} = L^A(c^\pi)$, $L^{B*} = L^B(c^\pi)$. From what has been established in Part III it follows that $\frac{dL^A}{dc^\pi} > 0$, $\frac{dL^B}{dc^\pi} > 0$ at the admissible values of c^π.

3. Finally, note that for fixed values of w, I, γ, l_1, l_1, H^π, LF and c_1^A, the value of the Gini index in the equilibrium position of the economy can be represented as a function of c^π:

$$G^* = G^*(c^\pi) = G\,[L^A(c^\pi), L^B(c^\pi)]$$

The total derivative of the function is

$$\frac{dG^*}{dc^\pi} = \frac{\partial G}{\partial L^A}\frac{dL^A}{dc^\pi} + \frac{\partial G}{\partial L^B}\frac{dL^B}{dc^\pi} \tag{IV.1}$$

Thus consider a first equilibrium position of the economy with $c^\pi = \bar{c}^\pi$ and such that the conditions (*C*) are fulfilled. What has been said in steps 1-2 concerning the sign of the derivatives in the right-hand side of (IV.1) implies that $\frac{dG^*}{dc^\pi} < 0$ for $c^\pi = \bar{c}^\pi$. Moreover, note that at any admissible value of c^π greater than $\bar{c}^\pi$ the conditions (*C*) continue to hold since $L^{B*} \geq H^\pi$ continues to hold. As a consequence, the argument put forward to assess the sign of $\frac{dG^*}{dc^\pi}$ for $c^\pi = \bar{c}^\pi$ can be repeated for $c^\pi \in (\bar{c}^\pi, 1)$. We thus conclude that $\frac{dG^*}{dc^\pi} < 0$ for $c^\pi \in [\bar{c}^\pi, 1)$, which means that any equilibrium position obtained from the first by attributing to c^π greater value than $\bar{c}^\pi$, while keeping all the other parameters unchanged, will be associated with a lower value of the Gini index.

V. Proof of proposition 4

Consider the solutions to *MV1*:

$$Y_1^* = \frac{l_1[c_1^A(w-\tau)+\gamma w]}{2-(w-\tau)l_1[1+c_1^A+(1-c_1^A)\gamma w l_1]}\left[\frac{c^\pi}{1-c^\pi}+\frac{(1-c_1^A)(w-\tau)l_3}{1-c_1^A(w-\tau)l_1}\right]I+\frac{c_1^A(w-\tau)l_3}{1-c_1^A(w-\tau)l_1}I$$

$$Y_2^* = \frac{2[1-c_1^A(w-\tau)l_1]}{2-(w-\tau)l_1[1+c_1^A+(1-c_1^A)\gamma w l_1]}\left[\frac{c^\pi}{1-c^\pi}+\frac{(1-c_1^A)(w-\tau)l_3}{1-c_1^A(w-\tau)l_1}\right]I$$

It clearly emerges from the first equation that Y_1^* falls as τ raises in the interval $[0, w-w^B)$. As regards Y_2^*, note that the second equation can be written

$$Y_2^* = A\,\frac{c^\pi I}{1-c^\pi}+B$$

with $A = \dfrac{2[1-c_1^A(w-\tau)l_1]}{2-(w-\tau)l_1[1+c_1^A+(1-c_1^A)\gamma w l_1]}$, $B = \dfrac{2(1-c_1^A)(w-\tau)l_3 I}{2-(w-\tau)l_1[1+c_1^A+(1-c_1^A)\gamma w l_1]}$

Moreover, note that:

$$\frac{dA}{d\tau} = \frac{2l_1[(c_1^A - 1) - (1 - c_1^A)\gamma w l_1]}{\{2 - (w - \tau)l_1[1 + c_1^A + (1 - c_1^A)\gamma w l_1]\}^2}$$

Since $0 < c_1^A < 1$ and γ, w, l_1 are all strictly positive, we readily conclude that $\frac{dA}{d\tau} < 0$. Considering that $\frac{c^\pi I}{1 - c^\pi}$ is a strictly positive constant and that $\frac{dB}{d\tau} < 0$ obviously holds, this means that also Y_2^* falls as τ raises in the interval $[0, w - w^B)$. As a consequence, both L^{A*} and L^{B*} must fall as τ raises from zero.

VI. Proof of Proposition 5

1. In the case of *MV1*, the formula for the Gini index can be written as

$$G = \frac{1}{N}\left[LF - l_1 \frac{\gamma w L^B(L^B + 2L^A + H^\pi) + \tilde{w}L^A(L^A + H^\pi)}{L^B + L^A + (l_1 - l_3)I}\right] \tag{VI.1}$$

where $\tilde{w} = w - \tau$.

2. For fixed values of w, I, γ, l_1, l_3, H^π and LF, (VI.1) yields the value of the Gini index as a well-defined function of L^A, L^B and $\tilde{w}$ that we denote $G(L^A, L^B, \tilde{w})$. The partial derivatives are:

$$\frac{\partial G}{\partial L^A} = -\frac{l_1}{N} \frac{[2(\gamma w L^B + \tilde{w}L^A) + \tilde{w}H^\pi](l_1 - l_3)I + \tilde{w}[(L^A)^2 + L^B(2L^A + H^\pi)] + \gamma w L^B(L^B - H^\pi)}{[L^B + L^A + (l_1 - l_3)I]^2}$$

$$\frac{\partial G}{\partial L^B} = -\frac{l_1}{N} \frac{\gamma w[(L^B)^2 + L^A H^\pi + (2L^B + 2L^A + H^\pi)(l_1 - l_3)I] + (2\gamma w L^B - \tilde{w}H^\pi)L^A + (2\gamma w - \tilde{w})(L^A)^2}{[L^B + L^A + (l_1 - l_3)I]^2}$$

$$\frac{\partial G}{\partial \tilde{w}} = -\frac{l_1}{N} \frac{L^A(L^A + H^\pi)}{L^B + L^A + (l_1 - l_3)I}$$

It is easily ascertained that for $l_1 \geq l_3$, $0.5 \leq \gamma < 1$, $L^B \geq H^\pi$ the following inequalities simultaneously hold:

$$\frac{\partial G}{\partial L^A} < 0, \quad \frac{\partial G}{\partial L^B} < 0, \quad \frac{\partial G}{\partial \tilde{w}} < 0 \tag{VI.2}$$

3. Now consider that for fixed values of w, I, γ, l_1, l_3, H^π, LF, c_1^A and c^π, the equilibrium employment levels L^{A*}, L^{B*} can be represented as functions of τ. We can accordingly write $L^{A*} = L^A(\tau)$, $L^{B*} = L^B(\tau)$. From what has been established in Part V it follows that:

$$\frac{dL^A}{d\tau} < 0, \quad \frac{dL^B}{d\tau} < 0 \qquad \text{for } \tau \in [0, w - w^B) \tag{VI.3}$$

4. Finally, note that for fixed values of w, I, γ, l_1, l_3, H^π, LF, c_1^A and c^π, the value of the Gini index in the equilibrium position of the economy can be represented as a function of τ:

$$G^* = G^*(\tau) = G\,[L^A(\tau), L^B(\tau), \tilde{w}(\tau)]$$

where $\tilde{w}(\tau) = w - \tau$. The total derivative is

$$\frac{dG^*}{d\tau} = \frac{\partial G}{\partial L^A}\frac{dL^A}{d\tau} + \frac{\partial G}{\partial L^B}\frac{dL^B}{d\tau} + \frac{\partial G}{\partial \tilde{w}}\frac{d\tilde{w}}{d\tau} \tag{VI.4}$$

Hence, consider an initial equilibrium position of the economy with $\tau = 0$ and such that the conditions (C') are fulfilled. As explained in the text, the third of these conditions ensures that $L^{B*} \geq H^\pi$ holds in that position, which in turn implies that the inequalities (VI.2) hold. Considering that the inequalities (VI.3) must also hold and that $\frac{d\tilde{w}}{d\tau} = -1$, we conclude that $\frac{dG^*}{d\tau} > 0$ in the initial equilibrium. Now note that, at any admissible positive value of τ, the third of conditions (C') ensures that $L^{B*} \geq H^\pi$ continues to hold in the corresponding new equilibrium and, for this reason, that the inequalities (VI.3) continue to hold. The argument previously developed for $\tau = 0$ can therefore be repeated to establish that the sign of $\frac{dG^*}{d\tau}$ remains strictly positive. We thus conclude that $\frac{dG^*}{d\tau} > 0$ for $\tau \in [0, w - w^B)$, which means that any new equilibrium position obtained from the initial one by simply attributing a positive value to τ will be associated with a higher value of the Gini index.

VII. Asymptotic convergence of *MV1*

1. By substituting into the equations (4')-(5) of *MV1*, and then taking the values of Y_1, Y_2 as functions of time, we obtain the following formulation of the excess demands for commodity 1 and commodity 2:

$$ED_1(t) = (c_1^A \tilde{w} l_1 - 1)Y_1(t) + (c_1^A \tilde{w} + w^B) l_2 Y_2(t) + c_1^A \tilde{w} l_3 I$$

$$ED_2(t) = [c^\pi + (c_2^A - c^\pi)\tilde{w} l_1]Y_1(t) + [c^\pi(1 - w^B l_2) + (c_2^A - c^\pi)\tilde{w} l_2 - 1]Y_2(t) + [c^\pi + (c_2^A - c^\pi)\tilde{w} l_3]I$$

where $\tilde{w} = w - \tau$. The adjustment process (*) mentioned in the text can therefore be represented through the following system of first-order, linear differential equations with constant coefficients:

$$\begin{bmatrix} Y_1'(t) \\ \\ Y_2'(t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} Y_1(t) \\ \\ Y_2(t) \end{bmatrix} + \begin{bmatrix} c_1^A \tilde{w} l_3 I \\ \\ [c^\pi + (c_2^A - c^\pi)\tilde{w} l_3] I \end{bmatrix} \tag{S}$$

where

$$\mathbf{A} = \begin{bmatrix} c_1^A \tilde{w} l_1 - 1 & (c_1^A \tilde{w} + w^B) l_2 \\ c^\pi + (c_2^A - c^\pi)\tilde{w} l_1 & c^\pi (1 - w^B l_2) + (c_2^A - c^\pi)\tilde{w} l_2 - 1 \end{bmatrix}$$

2. Now denote by Y_1^{**}, Y_2^{**} the outputs of industries 1 and 2 in the new equilibrium with $\tau > 0$. Since the functions $Y_1(t) = Y_1^{**}$, $Y_2(t) = Y_2^{**}$ constitute a particular solution to (S), to prove that the economy asymptotically converges to the new equilibrium amounts to proving that the general solution to the homogeneous system associated with (S) approaches the equilibrium point (0, 0) as $t \rightarrow \infty$. As a first step in the latter direction, we shall highlight three properties of the matrix **A**.

Property I: det(**A**) > 0. To verify this property, consider the matrix **A'** obtained from **A** by adding the first row to the second and then dividing the modified second row by $(c^\pi - 1)$:

$$\mathbf{A'} = \begin{bmatrix} c_1^A \tilde{w} l_1 - 1 & (c_1^A \tilde{w} + w^B) l_2 \\ 1 - \tilde{w} l_1 & 1 - (\tilde{w} + w^B) l_2 \end{bmatrix}$$

From basic rules for determinants, det(**A**) = $(c^\pi - 1)$det(**A'**). Moreover, the assumed conditions $1 - l_1 w > 0$, $w^B < \tilde{w} = w - \tau$, $l_2 = 0.5\, l_1$ imply that the elements in the diagonal of **A'** have opposite signs and that the off diagonal elements are strictly positive. As a consequence, det(**A'**)<0. From this result and the fact that $(c^\pi - 1) < 0$ by assumption, Property I immediately follows.

Property II: tr(**A**) < 0. By definition,

$$\mathrm{tr}(\mathbf{A}) = c_1^A \tilde{w} l_1 - 1 + c^\pi (1 - w^B l_2) + (c_2^A - c^\pi)\tilde{w} l_2 - 1 \qquad \text{(VII.1)}$$

which can equivalently be written as:

$$\mathrm{tr}(\mathbf{A}) = (2\tilde{w} l_2 - 1) - c_2^A \tilde{w} l_2 + (c^\pi - 1) - c^\pi (\tilde{w} + w^B) l_2 \qquad \text{(VII.2)}$$

Since the assumed conditions $1 - l_1 w > 0$, $l_2 = 0.5\, l_1$ imply that $(2\tilde{w} l_2 - 1) < 0$, we see from (VII.2) that Property II holds.

Property III: The characteristic equation associated with **A** has distinct real roots λ_1, λ_2. To verify this property it must be proved that $\Delta = [\mathrm{tr}(\mathbf{A})]^2 - 4\,\det(\mathbf{A}) > 0$, which is equivalent to proving that the product of the off diagonal elements of **A** is strictly positive (Sydsaeter, Hammond, 1995, p. 482). To prove the latter point, note preliminarily that $c^\pi + (c_2^A - c^\pi)\tilde{w} l_1 = c_2^A \tilde{w} l_1 + c^\pi (1 - \tilde{w} l_1) > 0$ since by assumption $1 - l_1 w > 0$. Inspection of **A** then shows that both the off diagonal elements are strictly positive.

3. Let us now use the first two properties to qualify the third. From basic results on square matrices, $\lambda_1 \lambda_2 = \det(\mathbf{A})$, $\lambda_1 + \lambda_2 = \mathrm{tr}(\mathbf{A})$ (e.g. Takayama, 1994, p. 598). Taking Properties I-II into account, this means that the distinct real roots λ_1, λ_2 are negative numbers, which in turn ensures that the solution to the homogeneous system associated with (S) asymptotically converges to the point (0, 0) (e.g. Takayama, 1994, p. 395).

VIII. Proof of proposition 6

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$$Y_1^* = \frac{l_1(c_1^A + \gamma w + \tau)}{2 - wl_1[1 + c_1^A + (1 - c_1^A)l_1(\gamma w + \tau)]} \left[\frac{c^\pi}{1 - c^\pi} + \frac{(1 - c_1^A)wl_3}{1 - c_1^A wl_1} \right] I + \frac{c_1^A wl_3}{1 - c_1^A wl_1} I$$

$$Y_2^* = \frac{2(1 - c_1^A wl_1)}{2 - wl_1[1 + c_1^A + (1 - c_1^A)l_1(\gamma w + \tau)]} \left[\frac{c^\pi}{1 - c^\pi} + \frac{(1 - c_1^A)wl_3}{1 - c_1^A wl_1} \right] I$$

It clearly appears from these equations that both Y_1^* and Y_2^* raise as τ raises in the interval $[0, w - w^B)$. As a consequence, both L^{A*} and L^{B*} must raise as τ raises from zero.

IX. Proof of Proposition 7

1. In the case of $MV2$, the formula for the Gini index can be written as:

$$G = \frac{1}{N} \left[LF - l_1 \frac{\tilde{w}^B L^B (L^B + 2L^A + H^\pi) + wL^A(L^A + H^\pi)}{L^B + L^A + (l_1 - l_3)I} \right] \qquad \text{(IX.1)}$$

where $\tilde{w}^B = \gamma w + \tau$.

2. For fixed values of w, I, l_1, l_3, H^π and LF, (IX.1) yields the value of the Gini index as a well-defined function of L^A, L^B, $\tilde{w}^B$ that we denote $G(L^A, L^B, \tilde{w}^B)$. The partial derivatives are:

$$\frac{\partial G}{\partial L^A} = -\frac{l_1}{N} \frac{[2(\tilde{w}^B L^B + wL^A) + wH^\pi](l_1 - l_3)I + \tilde{w}^B (L^B)^2 + wL^A(2L^B + L^A) + (w - \tilde{w}^B)L^B H^\pi}{[L^B + L^A + (l_1 - l_3)I]^2}$$

$$\frac{\partial G}{\partial L^B} = -\frac{l_1}{N} \frac{\tilde{w}^B[2(L^A + L^B) + H^\pi](l_1 - l_3)I + L^A[(2\tilde{w}^B L^B - wH^\pi) + (2\tilde{w}^B - w)L^A] + \tilde{w}^B[(L^B)^2 + L^A H^\pi]}{[L^B + L^A + (l_1 - l_3)I]^2}$$

$$\frac{\partial G}{\partial \tilde{w}^B} = -\frac{l_1}{N} \frac{L^B(L^B + 2L^A + H^\pi)}{L^B + L^A + (l_1 - l_3)I}$$

It is easily ascertained that for $l_1 \geq l_3$, $0.5 \leq \gamma < 1$, $L^B \geq H^\pi$ the following inequalities simultaneously hold:

$$\frac{\partial G}{\partial L^A} < 0, \ \frac{\partial G}{\partial L^B} < 0, \ \frac{\partial G}{\partial \tilde{w}^B} < 0 \qquad \text{(IX.2)}$$

3. Now consider that for fixed values of w, I, γ, l_1, l_3, H^π, LF, c_1^A and c^π, the equilibrium employment levels L^{A*}, L^{B*} can be represented as functions of the subsidy τ. We can accordingly write $L^{A*} = L^A(\tau)$, $L^{B*} = L^B(\tau)$. What has been established in Part VIII implies that the following inequalities simultaneously hold at the admissible values of τ:

$$\frac{dL^A}{d\tau} > 0, \ \frac{dL^B}{d\tau} > 0 \qquad \text{(IX.3)}$$

4. Finally, note that for fixed values of w, I, γ, l_1, l_3, H^π, LF, c_1^A and c^π, the value of the Gini index in the equilibrium position of the economy can be represented as a function of τ:

$$G^* = G^*(\tau) = G\,[L^A(\tau), L^B(\tau), \tilde{w}^B(\tau)]$$

where $\tilde{w}^B(\tau) = \gamma w + \tau$. The total derivative is:

$$\frac{dG^*}{d\tau} = \frac{\partial G}{\partial L^A}\frac{dL^A}{d\tau} + \frac{\partial G}{\partial L^B}\frac{dL^B}{d\tau} + \frac{\partial G}{\partial \tilde{w}^B}\frac{d\tilde{w}^B}{d\tau} \qquad \text{(IX.4)}$$

Thus consider an initial equilibrium position of the economy with $\tau = 0$ and such that the conditions (C) are fulfilled. Since the inequalities (IX.2)-(IX.3) must hold in that position, and since $\frac{d\tilde{w}^B}{d\tau} = 1$, we conclude that $\frac{dG^*}{d\tau} < 0$ in the initial equilibrium with $\tau = 0$. Then note that at any admissible positive value of τ, the conditions (C) continue to hold since $L^{B*} \geq H^\pi$ continues to hold. The argument put forward for $\tau = 0$ can accordingly be repeated to establish that $\frac{dG^*}{d\tau} < 0$ for $\tau \in (0, w - w^B)$. We thus conclude that $\frac{dG^*}{d\tau} < 0$ for $\tau \in [0, w - w^B)$, which means that any new equilibrium position obtained from the initial one by simply attributing a positive value to τ will be associated with a lower value of the Gini index.

X. Asymptotic convergence of *MV2*

1. By substituting into the equations (4”)-(5) of *MV2*, and then taking the values of Y_1, Y_2 as functions of time, we obtain the following formulation of the excess demands for commodity 1 and commodity 2:

$$ED_1(t) = (c_1^A w l_1 - 1)Y_1(t) + (c_1^A w + \tilde{w}^B) l_2 Y_2(t) + c_1^A w l_3 I$$

$$ED_2(t) = [c^\pi + (c_2^A - c^\pi) w l_1] Y_1(t) + [c^\pi (1 - \tilde{w}^B l_2) + (c_2^A - c^\pi) w l_2 - 1] Y_2(t) + [c^\pi + (c_2^A - c^\pi) w l_3] I$$

where $\tilde{w}^B = \gamma w + \tau$. The adjustment process (*) as applied to *MV2* can therefore be represented through the following system of differential equations:

$$\begin{bmatrix} Y_1'(t) \\ \\ Y_2'(t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} Y_1(t) \\ \\ Y_2(t) \end{bmatrix} + \begin{bmatrix} c_1^A w l_3 I \\ \\ [c^\pi + (c_2^A - c^\pi) w l_3] I \end{bmatrix} \qquad (S')$$

where

$$\mathbf{A} = \begin{bmatrix} c_1^A w l_1 - 1 & (c_1^A w + \tilde{w}^B) l_2 \\ \\ c^\pi + (c_2^A - c^\pi) w l_1 & c^\pi (1 - \tilde{w}^B l_2) + (c_2^A - c^\pi) w l_2 - 1 \end{bmatrix}$$

2. To prove that the economy asymptotically converges to the new equilibrium boils down to proving that the characteristic equation associated with **A** has distinct, real and negative roots. As a first step in the latter direction we shall highlight three properties of **A**.

Property I: det(**A**) > 0. Consider the matrix **A'** obtained from **A** by adding the first row to the second and then dividing the modified second row by $(c^\pi - 1)$:

$$\mathbf{A'} = \begin{bmatrix} c_1^A w l_1 - 1 & (c_1^A w + \tilde{w}^B) l_2 \\ \\ 1 - w l_1 & 1 - (w + \tilde{w}^B) l_2 \end{bmatrix}$$

By inspecting **A'** and taking the assumed conditions $1 - l_1 w > 0$, $\tilde{w}^B < w$, $l_2 = 0.5\, l_1$ into account, it is readily concluded that det(**A'**)<0. Since det(**A**) = $(c^\pi - 1)$ det(**A'**), this means that det(**A**)>0.

Property II: tr(**A**) < 0. By definition,

$$\mathrm{tr}(\mathbf{A}) = c_1^A w l_1 - 1 + c^\pi (1 - \tilde{w}^B l_2) + (c_2^A - c^\pi) w l_2 - 1 \qquad (X.1)$$

which can equivalently be written as

$$\mathrm{tr}(\mathbf{A}) = (2 w l_2 - 1) - c_2^A w l_2 + (c^\pi - 1) - c^\pi (w + \tilde{w}^B) l_2 \qquad (X.2)$$

Considering that $1 - l_1 w > 0$, $l_2 = 0.5\, l_1$ by assumption, we see from (X.2) that Property II holds.

Property III: The characteristic equation associated with **A** has distinct real roots λ_1, λ_2. As pointed out in Part VII, to demonstrate that this property holds amounts to proving that the product of the off diagonal elements of **A** is strictly positive. The latter point is easily proved by inspecting **A** and considering that $c^{\pi} + (c_2^A - c^{\pi})wl_1 = c^{\pi}(1 - wl_1) + c_2^A wl_1 > 0$ in view of the profitability condition $1 - l_1 w > 0$.

3. Finally, Properties I-II can be used as in Part VII to prove that the distinct real roots λ_1, λ_2 are negative numbers.